Project Management Under Risk: Using the Real Options Approach to Evaluate Flexibility in R&D

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Abstract

The real options framework has been proposed to understand the value of managerial flexibility in the context of uncertain R&D projects because it can capture asymmetric upside potentials. We introduce, in addition to the familiar real option of abandonment, the option of corrective action that management can take during the project. The intuition from options pricing theory is that higher uncertainty in project payoffs increases the real option value of managerial decision flexibility. This article develops a model bridging the gap between financial uncertainty, as considered in options pricing theory, and operational uncertainty, encountered as stochastic variability in R&D operations. At the operational level, the impact of higher uncertainty on the value of flexibility is not well understood. We identify five example types of operational variability, in market payoffs, project budgets, quality performance, market requirements, and project schedules. The model shows that an increase in operational variability may "average out" useable project payoff variability, based on which flexibility can be exercised. Thus, the real option value of managerial flexibility may be reduced. This result runs counter to established option pricing theory intuition and should, thus, contribute to a better risk management in R&D projects. It helps to guide managers as to when and when not efforts to delay commitments and maintain flexibility should be made in R&D projects.

Keywords: Real options, R&D projects, project evaluation, decision trees, stochastic dynamic programming, managerial flexibility, project management.

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*The authors are named in alphabetical order.

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1 Introduction and Literature Overview

Most investment decisions (and R&D investments in particular) are characterized by irreversibility and uncertainty about their future rewards: once money is spent, it can not be recovered if the payoffs hoped for do not materialize. However, a firm usually has some leeway over the *timing* of the investment: it has the right but not the obligation to buy an asset (e.g., access to a profitable market in the case of an R&D project) at some future time of its choosing, and thus, it is holding an option, analogous to a financial call option (Dixit and Pindyck 1994). As new information arrives and uncertainty about the investment's rewards is gradually resolved, management often has the flexibility to alter the initial operating strategy adopted for the investment. As with options on financial securities, this flexibility to adapt in response to new information enhances the investment opportunity's value by improving its upside potential while limiting downside losses relative to the initial expectations (Trigeorgis 1997). Using the analogy with options on financial assets, such investment flexibility is often called a "real option." Recognizing the option character of strategic investments allows evaluation of the opportunities offered by them (e.g., Kogut and Kulatilaka 1994).

This flexible decision structure of options is valid in an R&D context: after an initial investment, management can gather more information about the uncertain payoffs and, based on this information, change its course of action (make a contingent decision, see, e.g., Dixit and Pindyck 1994, Lint and Pennings 1997). The real options approach can evaluate the project value "correctly" (a pure net present value analysis understates the project value), as well as estimate the value of managerial flexibility itself. Five basic sources of flexibility (or option value) have been distinguished (e.g., Trigeorgis 1997): A *defer option* refers to the possibility of waiting until more information has become available. An *abandonment option* is the possibility to make the investment in stages, deciding at each stage, based on the newest information, whether to proceed further or whether to stop (note the similarity to venture capital approaches). An *expansion or contraction option* represents the possibility to adjust the scale of the investment (e.g., a production facility) depending on whether market conditions turn out favorably or not. Finally, a *switching option* is the flexibility to switch operation of the asset from one mode to another, depending on factor prices (e.g., switching the energy source of a power plant, or switching raw material suppliers).

One key insight from the real options approach to investment is that *higher uncertainty in the payoffs of the investment increases the value of managerial flexibility, or the value of the real option* (Dixit and Pindyck 1994, p. 11; this was also shown by Roberts and Weitzman (1981) in a sequential decision model without referring to real options at all). The intuition is clear - with higher payoff uncertainty, flexibility has a higher potential of enhancing the upside while limiting the downside. An important managerial implication of this insight is that the more uncertain the project payoff is, the more efforts should be made to delay commitments and maintain the flexibility to change the course of action. This intuition is appealing. Yet, there is hardly any evidence of real options pricing of R&D projects in practice (see Smith and McCardle 1998; this is confirmed to us by conversations with R&D managers) despite reports that Merck uses the method (Sender 1994). Moreover, there are recent results presenting evidence that more uncertainty may *reduce* the option value if an alternative "safe" project is available (Kandel and Pearson 1998).

We view this evidence as a gap between the financial payoff variability, as addressed by the real options pricing literature, and operational uncertainty faced at the level of R&D management. For example, R&D project managers encounter uncertainty about budgets, schedules, technical performance, or market requirements, in addition to financial payoffs. The relationship between such operational uncertainty and the value of managerial flexibility (option value of the project) is not clear. For example, should the manager respond to increased uncertainty about technical product performance by delaying commitments, analogously to uncertainty about project payoffs? The first contribution of this article lies in connecting these operational sources of uncertainty to payoff uncertainty and, thus, the real option value of managerial flexibility. In a simple model, we demonstrate that operational uncertainty (in particular, uncertainty in product quality, market performance requirements and schedule adherence) may *reduce* the real option value. We interpret this counter-intuitive result in terms of the impact operational variability has on the useable information about uncertain payoffs: if operational variability increases the variability of final project payoffs and incoming information based on which flexibility can be applied, it improves management's ability to protect itself against a downside and, therefore, enhances the option value of managerial flexibility. However, if operational variability destroys available information about the project payoffs, it reduces the ability to respond and, thus, diminishes the option value of flexibility.

As a second contribution, we extend the usual taxonomy of types of real options (delay, abandon, contract, expand, switch) by "improvement." Mid-course actions during R&D projects to improve the technical performance of the product (or to correct its targeting to market needs) are commonly used. The availability of such improvement actions represent an additional source of option value.

The literature on real options is quite large – readers are referred to textbooks such as Dixit and Pindyck (1994), Copeland et al. (1995) Trigeorgis (1997), or Hull (1997) for overviews. Most applications of real options have been in the area of commodities (such as oil exploration) because in this environment, the financial markets are well developed and allow to replicate risks by traded assets. Recently, research has been carried out on the application of real options pricing to R&D projects (e.g., Brennan and Schwartz 1985, Faulkner 1996, McDonald and Siegel 1985, Mitchell and Hamilton 1988, Teisberg 1994).

2 Five Types of Operational Uncertainty

Figure 1 shows a simple conceptual picture of the drivers of project value: a new product development project is characterized by the standard performance measures cost, time, and quality. The market is characterized by its performance requirements and payoff potential (e.g., market size and attractiveness). Project and market characteristics together determine the total project benefits, or the value of the project to the firm. An important insight from real options theory is that more uncertainty in the project value *enhances* the option value of managerial flexibility because the very nature of an option consists in eliminating the downside while allowing to enjoy the benefits of the upside. The question we examine in this article is: does this insight hold as well for uncertainty in the operational value drivers shown in Figure 1? Each of the five drivers is typically characterized by uncertainty. We represent uncertainty by *stochastic variability* of parameter distributions, and in the remainder of this article, we use uncertainty and (stochastic) variability interchangeably.

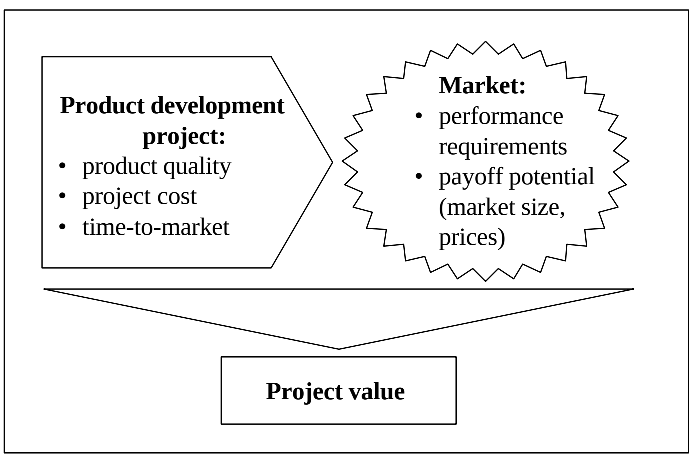

Figure 1: Five Types of Operational Variability

1. *Market Payoff Variability.* The market payoff potential depends on uncontrollable factors such as competitor moves, demographic changes, substitute products, etc. It is, therefore, common that sales forecasts have a significant random (unforeseeable)

component (even if this random component is sometimes admitted only implicitly).

2. *Budget Variability.* It refers to the fact that the running development costs of the project are not entirely foreseeable. Budget overruns are common, and less often, under-budget completion also occurs.

3. *Quality Variability.* This corresponds to uncertainty in the technical performance of the product developed. Actual performance achieved can often not be fully predicted at the beginning of the project, as tradeoffs must be resolved among multiple technical criteria, which together determine performance in the customer's eye. The higher the technical novelty of a product, the higher is this uncertainty (e.g, Roussel et al. 1991).

4. *Market Requirement Variability.* This corresponds to uncertainty about the performance level required by the market. Especially for conceptually new products, the performance targets for a product are often only imperfectly known (e.g., Chandy and Tellis 1998, O'Connor 1998).

5. *Schedule Variability.* It corresponds to the project not finishing deterministically at the planned time, but being delayed. A delay in time-to-market may result in reduced market payoffs (via reduced market share or prices), as empirical work in product development shows (e.g., Datar et al. 1997).

The influence of variability in these operational drivers, as opposed to variability in project value, on the value of managerial flexibility has not been examined. It is important to understand the impact of operational drivers because often, different individuals in an organization are responsible for the different drivers. For example, a project manager may control cost, time, and quality of the project, while a marketing manager may be in charge of understanding and influencing performance requirements and payoff potential in the market. It is important for them to understand in which cases managerial flexibility creates value. Only then is it worth postponing commitments to maintain flexibility. After setting up our basic model in Section 3, we show in Section 4.1 and 4.2 that increased variability in market payoffs as well as in budgets may indeed enhance the option value of managerial flexibility, consistent with option pricing theory. The other types of operational variability, however, may have the effect of *reducing* the value of flexibility, as we show in the remaining subsections 4.3 - 4.5.

3 The Basic Model

The real option value of managerial flexibility can be evaluated using contingent claims analysis, developed for pricing options in financial markets. This approach requires, however, a complete market of risky assets capable of *exactly*[1] replicating the project's risk by the stochastic component of some traded asset (Dixit and Pindyck 1994, p. 121). Such replicability often does not apply for R&D projects, whose risks are typically idiosyncratic and uncorrelated with the financial markets. However, an equivalent approach to option evaluation is dynamic programming (Dixit and Pindyck 1994, p. 7, Smith and Nau 1995), which does not require asset replication. Thus, we develop in this section a dynamic programming model of an R&D investment.[2]

The drawback of the dynamic programming approach is that it does not address the question of the correct risk-adjusted discount rate. Dynamic programming requires an exogenously specified discount rate that reflects the decision maker's risk attitude. However, the risk of an R&D project is typically due to factors unique to this project and thus *unsystematic* or *diversifiable*. Therefore, a rational investor can diversify the project risk away by holding a portfolio of securities and does not require a risk premium. A reasonable

[1] Here, "exactly" means for every sample path of the realization of the uncertainty.

[2] Smith and McCardle 1998 propose an "integrated" approach for oil exploration projects, where they use option pricing for risks that can be replicated in the market and dynamic progamming for risks that cannot be priced.

assumption for a large firm is, therefore, a risk-neutral attitude toward the project with discounting at the risk-free rate (Trigeorgis 1997, p. 43).

Consider an R&D project proceeding in T discrete stages toward market introduction. The market success is determined by the product performance, which is modeled by a one-dimensional parameter i, such as processor speed in a computer, or in the case of a multi-attribute product, the customer utility derived from a conjoint analysis (see, e.g., Aaker and Day 1990). The project is subject to uncertainty stemming from the market and from technical development risk. Uncertainty of a model parameter manifests itself in the *variability* of a probability distribution. A distribution is said to exhibit higher variability than a second distribution if both have the same mean and the former has a higher variance. This definition corresponds to Rothschild and Stiglitz's (1970) definition of higher risk. Focusing on variability separates changes in distribution means from changes in risk.

Technical variability causes the product performance i to "drift" in every period, or stage, of the project. The state of the system is characterized by (i, t), the level of product performance i at time t. The drift follows a binomial distribution in each period, independent of the previous history of project progress. From period t to the next period, the performance may unexpectedly improve with probability p, or it may deteriorate with probability $(1-p)$ due to unexpected adverse events. We generalize the binomial distribution by allowing the performance improvement and deterioration, respectively, to be "spread" over the next N performance states with transition probabilities.

$$p_{ij} = \begin{cases} \frac{p}{N} & \text{if } j \in \{i+\frac{1}{2}, \ldots, i+\frac{N}{2}\} \\ \frac{1-p}{N} & \text{if } j \in \{i-\frac{1}{2}, \ldots, i-\frac{N}{2}\} \\ 0 & \text{otherwise} \end{cases} \tag{1}$$

The mean of this distribution is $\frac{N+1}{4}(2p-1)$, and the variance is $\frac{N+1}{8}[\frac{N}{3} + (N+1)(\frac{1}{3} - \frac{(2p-1)^2}{2})]$. With two parameters, this discrete distribution can approximate the first two moments of a range of continuous distributions. Moreover, this approximation leads to a recombining lattice model, which reduces the size of the state space and, thus, computational complexity. If $p = 0.5$ (a particularly relevant case for the analysis below), N characterizes the variability of the technical performance. The state space of technical performance over time is illustrated in Figure 2. Its left-hand side corresponds to the transition probabilities (1).

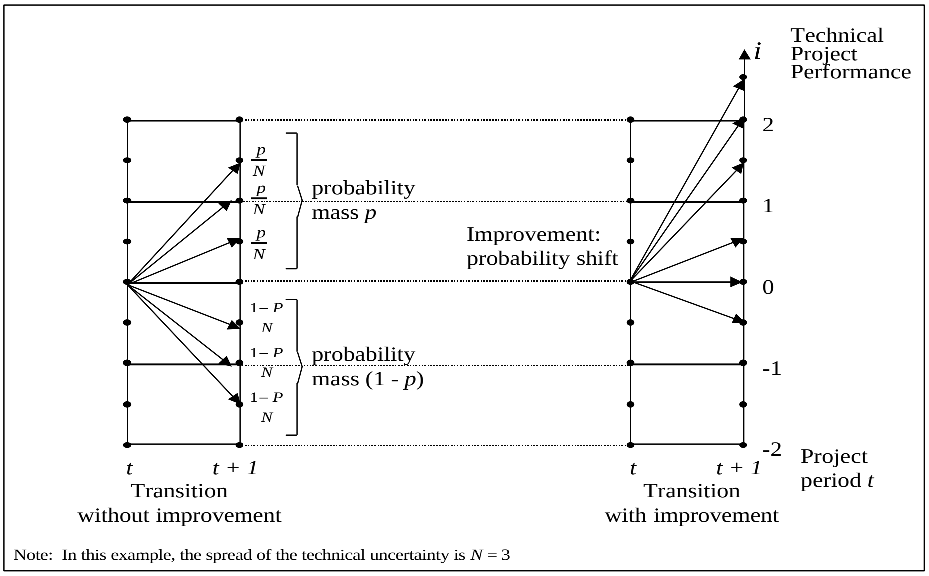

Figure 2: Transition Probabilities of Technical Performance

If $p = 0.5$, the expected technical performance state at launch is normalized at $Ei = 0$, which means the project plan is unbiased. If $p > 0.5$, the project plan is too "pessimistic", and the true expected performance at launch is $Ei > 0$. If $p < 0.5$, the project plan is too "optimistic", and the true expected performance is $Ei < 0$.

At each project stage t, management can take any one of three possible actions: abandon, continue, or improve. The first two options are standard in real option theory. Abandonment terminates the project immediately, foregoing any further costs or revenues. Continuation proceeds to the next stage $(t + 1)$ at a continuation cost of $c(t)$. The continuation cost usually increases over time for R&D projects: $c(t) \leq c(t + 1)$, but this is not required for our results. Over the period, the performance state evolves according to the transition probabilities shown above.

In addition to these two possibilities, management can also choose to take corrective action and inject additional resources to improve product performance in expectation by one level. This imposes an improvement cost of $\alpha(t)$ in addition to the continuation costs. The improvement cost, like the continuation cost, typically also increases over time since changes become more difficult as more of the product design is completed (again, this is not required for our results): $\alpha(t) \leq \alpha(t+1)$. The improvement results in a "mean shift" of the transition probabilities (right-hand side in Figure 1)[3].

$$p_{ij} = \begin{cases} \frac{p}{N} & \text{if } j \in \{i+1+\frac{1}{2}, \ldots, i+1+\frac{N}{2}\} \\ \frac{1-p}{N} & \text{if } j \in \{i+1-\frac{1}{2}, \ldots, i+1-\frac{N}{2}\} \\ 0 & \text{otherwise} \end{cases} \tag{2}$$

At the start of the project, an initial investment of I must be made, for example, to put the project infrastructure in place. Costs and revenues are discounted at the risk-free rate r, consistent with risk-neutrality of the firm with respect to idiosyncratic risk (as was discussed above). At each stage, project continuation and improvement costs have to be paid at the beginning of the period.

When the project is launched at time T with a performance level i, it will generate an *expected* market payoff Π_i in the form of an S-curve, that is, Π_i is convex-concave in i. This is a general market payoff model, which includes linear, convex or concave payoff functions as special cases (left-hand side in Figure 3, see Kalyanaram and Krishnan 1997, or Bhattacharya et al. 1998). Such an S-shaped payoff function can, for example, be the result of a competitive performance threshhold that is not known in advance. In such a case, the market requires a certain level of performance D, dictated by competitive dynamics. If the project meets or exceeds this performance level, the market will yield

[3]Note that this implies the improvement can be carried out purely with additional resources (such as engineers, or experimental lab capacity), without an additional time delay. Time delays, or schedule risk, will be treated separately in Section 4.5

a premium profit margin M. But if the project misses the target, it must compete on cost, and produces only a smaller margin m (right-hand side in Figure 3). The required market performance is not known to the firm in advance and is resolved only after the product launch. The firm has an efficient forecast in the form of a probability distribution F (center in Figure 3).

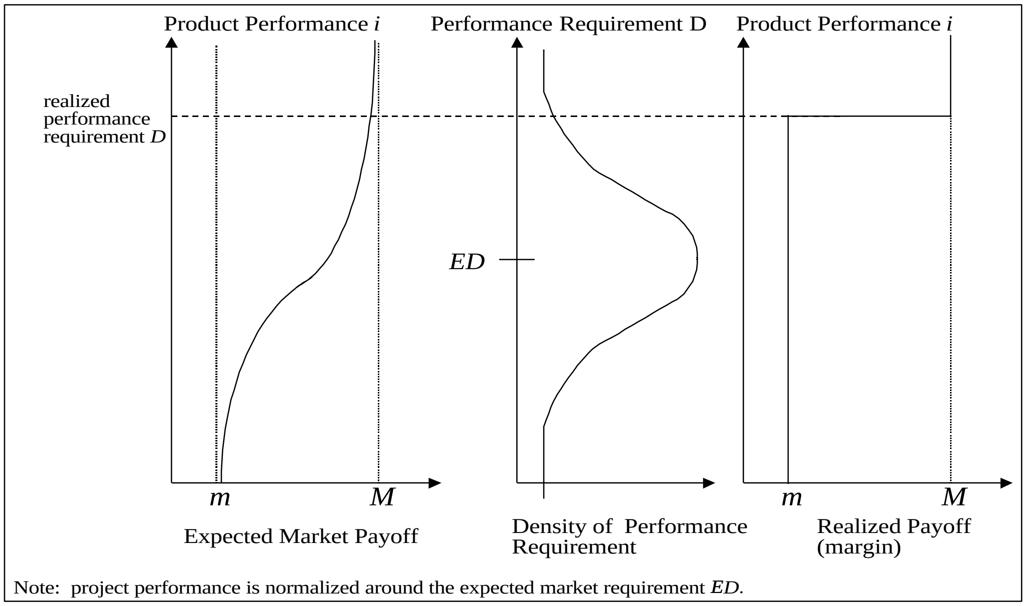

Figure 3: Market Uncertainty and Project Payoffs

Thus, if the project launches a product of performance level i, the *expected* payoff can be written as $\Pi_i = m + F(i)(M - m)$, where $F(i)$ represents the probability that i exceeds the realized performance requirement D. It can easily be shown that for any distribution F with a density that has a single maximum, Π_i is strictly convex-concave increasing in i.[4] For simplicity of exposition, we assume that the mode of F (that is the point where Π turns from convex to concave) is at the expectation ED.[5]

The sequential decision problem resulting from the above described set-up can be formu-

[4]For example, Kalyanaram and Krishnan (1997) use an equivalent setup, expressed in terms of number of customers asked who like the design, where F is assumed to be a normal distribution.

[5]This is true, for example, for the normal distribution. Our results are slightly simplified by this assumption, but do not depend on it.

lated as a dynamic program with the following value function:

$$V_i(T) = max \begin{cases} \text{abandon:} & 0; \\ \text{continue:} & -c(T) + \frac{\sum_{j=1}^{N}[p\Pi_{i+j/2}+(1-p)\Pi_{i-j/2}]}{N(1+r)}; \\ \text{improve:} & -c(T) - \alpha(T) + \frac{\sum_{j=1}^{N}[p\Pi_{i+1+j/2}+(1-p)\Pi_{i+1-j/2}]}{N(1+r)} \end{cases} \quad (3)$$

$$V_i(t) = max \begin{cases} \text{abandon:} & 0; \\ \text{continue:} & -c(t) + \frac{\sum_{j=1}^{N}[pV_{i+j/2}(t+1)+(1-p)V_{i-j/2}(t+1)]}{N(1+r)}; \\ \text{improve:} & -c(t) - \alpha(t) + \frac{\sum_{j=1}^{N}[pV_{i+1+j/2}(t+1)+(1-p)V_{i+1-j/2}(t+1)]}{N(1+r)} \end{cases} \quad (4)$$

We can characterize the optimal decision rule, or policy, for this dynamic program. Proposition 1 describes the optimal policy for an increasing and convex-concave Π_i (this includes as special cases Π_i convex or concave).

Proposition 1 *If the payoff function Π_i is convex-concave increasing, the optimal policy in period t is characterized by control limits $L_u(t) \geq L_m(t)$ and $L_d(t)$ (where all may be outside the range $[-Nt/2, (N/2+1)t]$) such that it is optimal to:*

- *choose abandonment if $L_d(t) \geq i$.*
- *Otherwise: choose continuation if $i > L_u(t)$,*
  choose improvement if $L_u(t) \geq i > L_m(t)$,
  and choose continuation if $L_m(t) \geq i$.

Moreover, the optimal value function $V(t)$ is also convex-concave increasing in i, and $L_u(t)$ lies in its concave region and $L_m(t)$ in its convex region.

Proof

For easier readability of the text, all proofs are shown in the appendix.

Figure 4 demonstrates the structure of the optimal policy. In the center where the convex-concave payoff-function is steepest, improvement is worthwhile. However, in the flatter regions of the payoff function, the higher payoff does not justify the improvement cost α_T. The lower control L_d cuts off the project whenever the expected payoff (over the $2N$ reachable states) is too low to justify the continuation cost c_T. If Π_i is concave, $L_m = -\infty$, and if Π_i is convex, $L_u = \infty$.

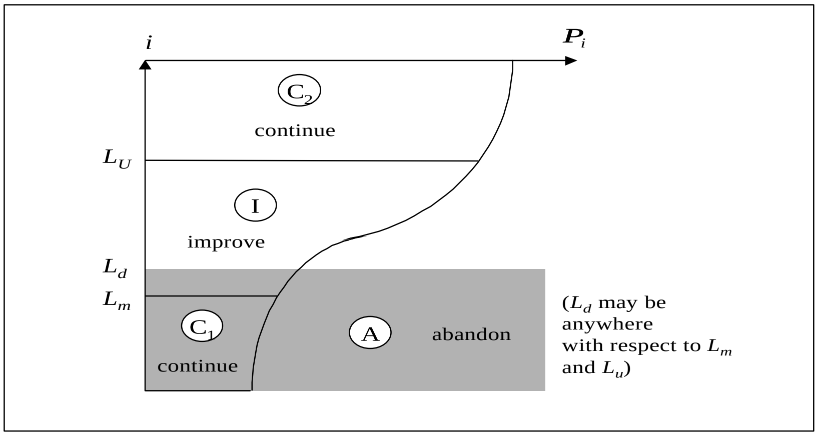

Figure 4: Control Limits of Optimal Policy

Figure 5 demonstrates the policy and the value function on an example. To the right is the market payoff function Π_i. The cone, corresponding to the increasing number of possible states over time, contains the values of the optimal value function. Below each state in the cone, the optimal value function along with the corresponding decision is shown. In this example, the uncertainty N has been set at 1 for easier exhibition. $V_0(t = 0)$ corresponds to the optimal value of the project before the investment costs of $I = 50$ are deducted. Below the cone, the optimal project value after deducting I is shown. This optimal project value includes the value of a *compound real option*, namely, the value of the managerial flexibility to choose improvement *or* abandonment (in addition to continuation) at each stage.

Along with the optimal project value, two benchmark values are shown: first, the project value resulting from having the possibility to abandon, but not to improve, in each period.

It comes as no surprise that it is lower than the optimal value, as it includes an abandonment option only, without the compounded improvement option. The second benchmark is the "traditional" net present value (NPV), which corresponds to setting all decisions equal to "C" (continue) and deciding at the beginning to do the project or not, depending on whether $V_0(0)$ exceeds I. If the project plan is unbiased, this is equivalent to discounting the expectation of the payoffs minus the appropriately discounted continuation costs. In this case, $V_0(0)$ corresponds to the project's NPV. At the bottom, finally, the option value itself, or the value of the managerial flexibility (to improve or to abandon), is shown. It corresponds to the difference between the optimal project value and the NPV.

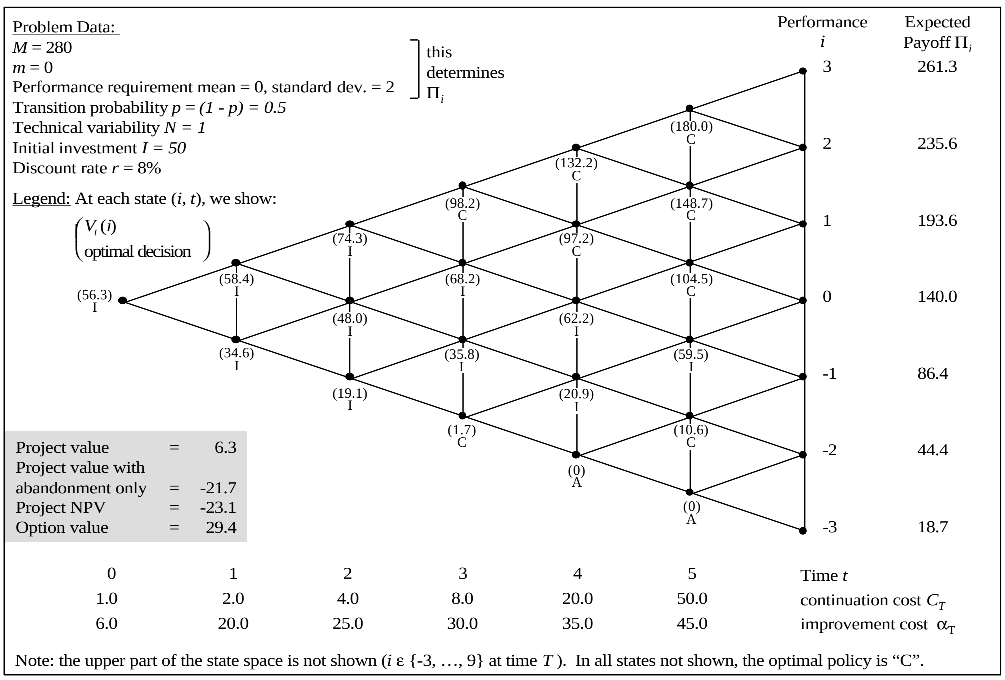

Figure 5: Example of Optimal Policy and Value Function

Improvement represents a different source of managerial flexibility, in addition to the standard "expansion" or "contraction" options. The operational character of a mid-course correction is different from expansion and contraction, which represent additional investment projects in their own right. Mid-course improvement during an R&D project, in contrast, represents an ability to reposition the project while it is ongoing. Expansion and contraction are strategic options. Mid-course improvement poses challenges for project management and for the operational process of producing the asset.

4 How Operational Uncertainty Influences the Option Value of Flexibility

4.1 Market Payoff Variability

In the context of our model, payoff variability corresponds to the difference $(M - m)$, holding the average constant. Suppose we have two financial payoff functions Π_i and $\overline{\Pi}_i$, both are convex-concave increasing, and $\overline{\Pi}_i$ exhibits greater variability in the Rothschild-Stiglitz (1970) sense: $\overline{\Pi}_0 = \Pi_0$, and $\overline{\Pi}_i - \Pi_i = \Pi_{-i} - \overline{\Pi}_{-i} \geq 0$ for all i.

Proposition 2 *Assume the project plan is unbiased, that is, $p = 0.5$. Then the option value corresponding to the payoff function with larger variability, $\overline{\Pi}_i$, is larger: $\overline{V}_0(0) \geq V_0(0)$ while the NPV remains unchanged.*

The reader may note that this proposition is valid in the case that Π_i is convex, convex-concave, or concave (that is, the result is independent of where the expected performance requirement ED is relative to the expected product performance $Ei = 0$). The condition that the project plan be unbiased serves to separate variance from mean effects. Consider the case of an "optimistic" project plan with $p < 0.5$. Then product quality will "drift" downward over time as the project progresses, and the payoff will be biased toward lower values. If the payoff function has higher variability, the project is likely to end up in the lower half of the performance range where the expected payoff *decreases* with the higher variability. In other words, the mean project value is likely to suffer. The options of improvement and abandonment may or may not suffice to offset this suffering of the mean payoff. If the project plan is "pessimistic" $(p > 0.5)$, product performance is biased toward the upper end of the performance range, and even the straight NPV already benefits from a payoff variability increase. Thus, the unbiased case that we analyze in Proposition 2 is the limit case where the NPV is not affected by the increase in variability.[6]

We have, thus, established that the fundamental option value of managerial flexibility exists in our model, that is, flexibility enhances the upside of the project while limiting the downside.

4.2 Budget Variability

Budget varability is already included in the model to the extent that improvement, the occurrence of which is stochastic according to the optimal policy, carries a cost α_t. The question is how the option value of improvement and abandonment is impacted if both improvement cost and continuation costs become stochastic, independent of whether improvement is chosen or not. Variability is represented by the variance θ^2 of the continuation cost c_t.

We need to consider two cases. First, if the c_t and α_t are independent of one another and over time, the optimal policy continues to hold, and both $V_0(0)$ and the NPV are unchanged. Thus, the option value of abandonment and improvement is unaffected, although the *variance* of the project payoff increases. The reason is that past expenses are "sunk," and past variations of the project costs carry no information about the future. Thus, the value of flexibility is not enhanced.

This changes if project costs are *correlated* over time, that is, if a budget overrun in c_t makes a future budget overrun more likely. In this case, the realization of c_t carries information

[6] The requirement that the two payoff functions cross at $i = 0$ and their differences are symmetric is required to ensure that the NPV for both is the same, which makes the change in $V_0(0)$ equal to the change in option value. Even if the two payoff functions have non-symmetric differences, the ideas described here remain valid, although exposition becomes more complicated because the change in NPV has to be factored into the analysis.

about the future, based on which flexibility can be used to improve the expected payoff. Formally, we can use the realization of c_t to update our estimate of the future value function. Suppose that c_t encapsulates all information from previous costs. Then we can expand the state space to (t, i, c_t), and the value function becomes $V(t, i, c_{t-1}) = E[c_t \mid c_{t-1}] + E[V(t+1) \mid c_{t-1}]$. The structure of the dynamic program remains. In this case, a higher variance θ of the continuation costs implies a higher variance of the (updated) $V(t+1)$, which by Proposition 2 means that the option value of flexibility increases. We conclude that the real option intuition continues to hold for the case of budget variability.

4.3 Quality Variability

In the context of our model, product quality corresponds to the technical performance i of the product, which varies stochastically because of the state transitions from one period to the next. Quality variability increases with parameter N and, thus, the variance in the transition probabilities (a larger N makes a larger number of states reachable in a transition). We now show that quality variability may *reduce* financial payoff variability and thus the real option value.

Proposition 3 *Assume the project plan is unbiased, that is, $p = 0.5$, and the expected performance requirement $ED = Ei = 0$. Then the option value $V_0(0)$ decreases when the quality variability N increases.*

The negative impact on the option value stems from the higher uncertainty "smearing out," or averaging out, the achievable performance over a wider range. This "smearing out" reduces the available payoff variability. The intuition is represented in Figure 6. From any current performance state during the project, higher technical uncertainty increases the range of possible payoff values. Thus, the expected payoff function flattens, which reduces the downside protection the decision maker can achieve by intelligently choosing improvement or abandonment of the project. In other words, higher quality uncertainty increases the possible performance range at launch. Therefore, the value of flexibility in response to this information suffers.

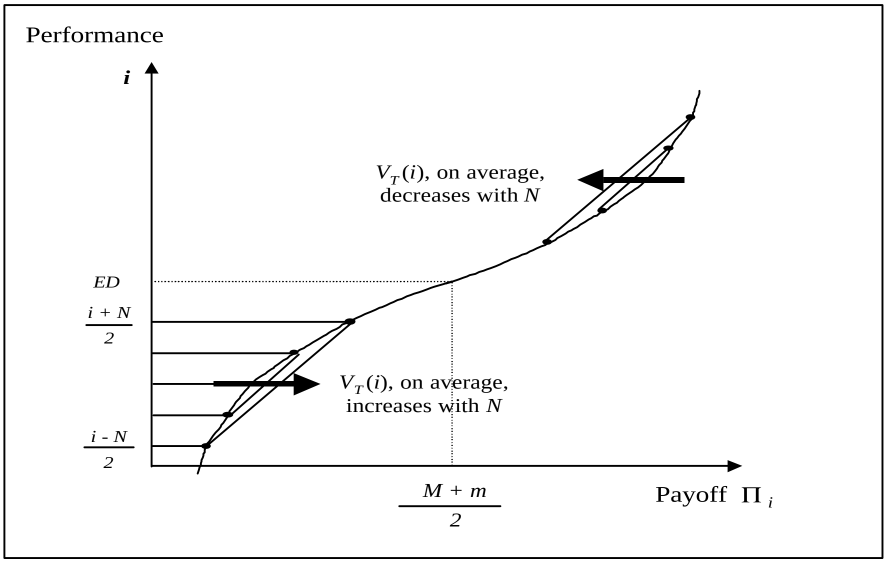

Figure 6: The Effect of Larger Quality Variability

This effect of quality variability does not appear if the payoff function Π_i is *linear* in the stochastic variable – e.g., the option value follows directly the stochastically varying poject performance. The convex-concavity in our model stems from the fact that the performance requirement in the market itself is stochastic (unforeseeable). Convex-concavity is the essential driver of our result that quality variability "washes out" payoff variability. Proposition 3, therefore, points to one important effect that needs to be included in the literature on real options, namely a non-linear relationship between an underlying stochastic variable and payoffs.

As before, this result has been "isolated" from other effects by assuming an unbiased project plan. If the project plan is "optimistic", i.e., $p < 0.5$, an increase in the quality variability parameter N shifts the expected project performance downward, making improvement and abandonment even more important and thus boosting the option value for an increasing N.[7] This is illustrated on the numerical example from Figure 5. If N is increased from 1 to 2, the option value decreases from 29.4 to 17.5. If, however, the upward transition probability is reduced to $p = 0.1$, the option value grows to 43 for $N = 1$ and even higher to 53 for $N = 2$.

4.4 Market Requirements Variability

In the context of our model, market requirement variability is represented by the variance σ^2 of the market performance requirement, while holding the mean market requirement ED constant. Proposition 4 shows another negative effect of operational variability on the option value.

Proposition 4 *Assume the project plan is unbiased, that is,* $p = 0.5$. *Then the option value* $V_0(0)$ *decreases if* σ, *the market requirement variability, increases. Furthermore, if* $V_0(0) \geq 0$ *for any* σ, *then there is a* $\overline{\sigma}$ *such that for all* $\sigma \geq \overline{\sigma}$ *the optimal policy is to "continue" in all states* (i, t), *in which case* $V_0(0) = NPV$, *the net present value.*

What is the reason for the real option value to be diminished by market requirement variability? The reason is summarized in Figure 7. When market requirements are more spread out without a corresponding increase in payoff variability, part of the probability mass "escapes" beyond the performance range in reach of the development project. As a result, the information about payoff variability offered by the current performance state i is reduced, which destroys the value of flexibly responding to this information. When variability becomes so great that in expectation no payoff difference exists over the reachable

[7] Similarly, if the expected market requirement $ED < 0$, the expected performance is larger than the expected requirement, in which case more of the distribution of i lies in the concave region of Π. In this case, $V_0(0)$ and the NPV both decrease, so the option value may increase or decrease. The converse holds if $ED > 0$. As in Proposition 2, the assumption in the proposition separates the variability effect on the option value from the mean effect on the NPV.

performance range, there remains no option benefit. The project decision becomes static since the performance states carry no information about payoffs. This static decision is equivalent to a NPV analysis without abandonment or improvement between project start and finish.

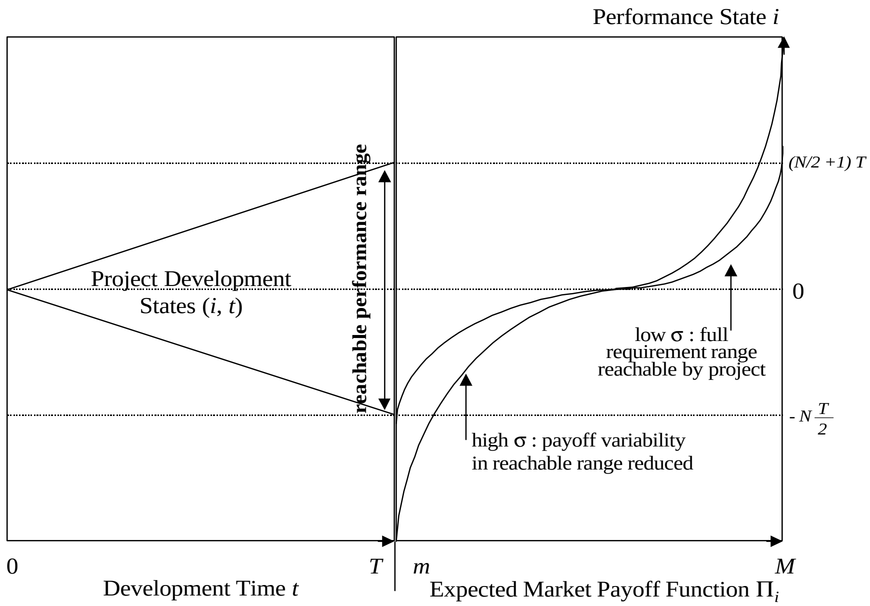

Figure 7: The Effect of Increased Requirements Variability

The decrease in option value is demonstrated in Figure 8 on the same example as in Figure 5 with market requirement variability increased from $\sigma = 2$ to $\sigma = 3$. The NPV value of the project has remained unchanged, but the value of both the abandonment option and the improvement option has been reduced. This becomes apparent when comparing the optimal policies between Figures 4 and 7. The number of states in which it is worthwhile to choose improvement has shrunk because the payoff function is flatter.

It is important to note that the reason for the lost option value in Proposition 4 is very different from that in Proposition 3. The effect in Proposition 4 has nothing to do with payoff nonlinearity (or with convex-concavity, for that matter). Indeed, the effect would persist with a linear function Π_i, which would be "rotated" around $i = 0$ such that its extreme values in the reachable performance cone would be pushed closer together. The

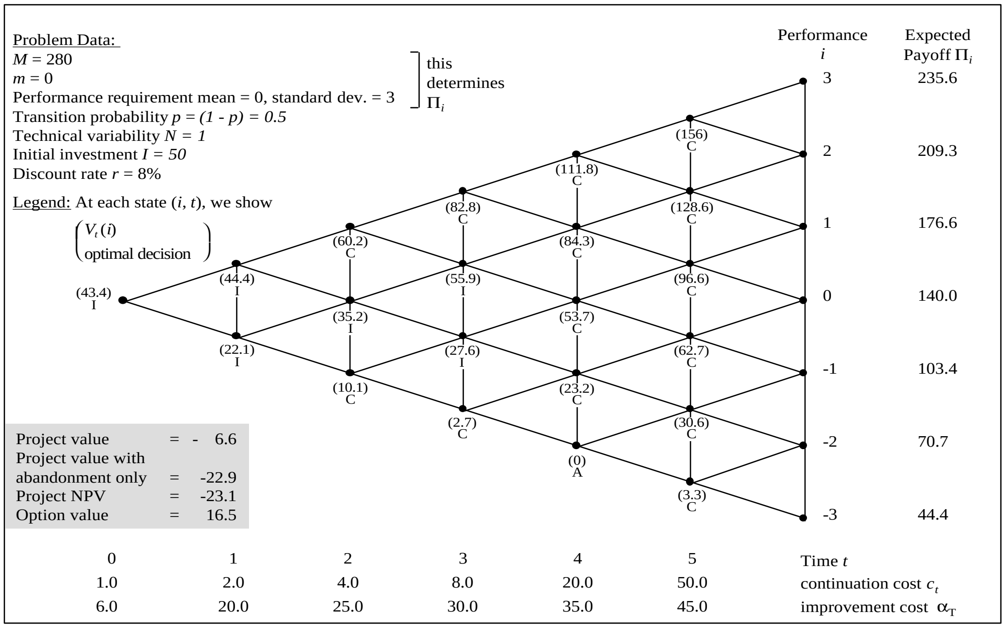

Figure 8: Example With Increased Requirements Variability

key phenomenon is that the endpoint of the payoff distribution is pushed beyond the reachable performance range and therefore, the *reachable* payoff variability is reduced.

This effect of probability mass escaping beyond a reachable "capacity" limit is very important in a different context as well. Consider an investment in a flexible production facility with a capacity limit. More variability can be detrimental if probability mass of demand, and thus part of the upside of the option, escapes beyond the capacity limit (see also Jordan and Graves 1995).

4.5 Schedule Variability

In this subsection, we discuss the impact of schedule variability on the option value of managerial flexibility. Suppose that the expected market payoff Π_i is sensitive to the time-to-market, that is, if the product launch is delayed by δ beyond the planned launch time T, $\Pi_i(\delta)$ is reduced. This is consistent with empirical results that a time-to-market delay may destroy development project payoffs, see, e.g., Datar et al. 1997. In order to focus on schedule variability and to make its effects very clear, we simplify our basic model by collapsing the technical performance states, i.e., by considering a situation where the target performance is well-known and reachable.[8] So, consider a two-stage project, in which stage 1 may be delayed, as is shown in Figure 9.

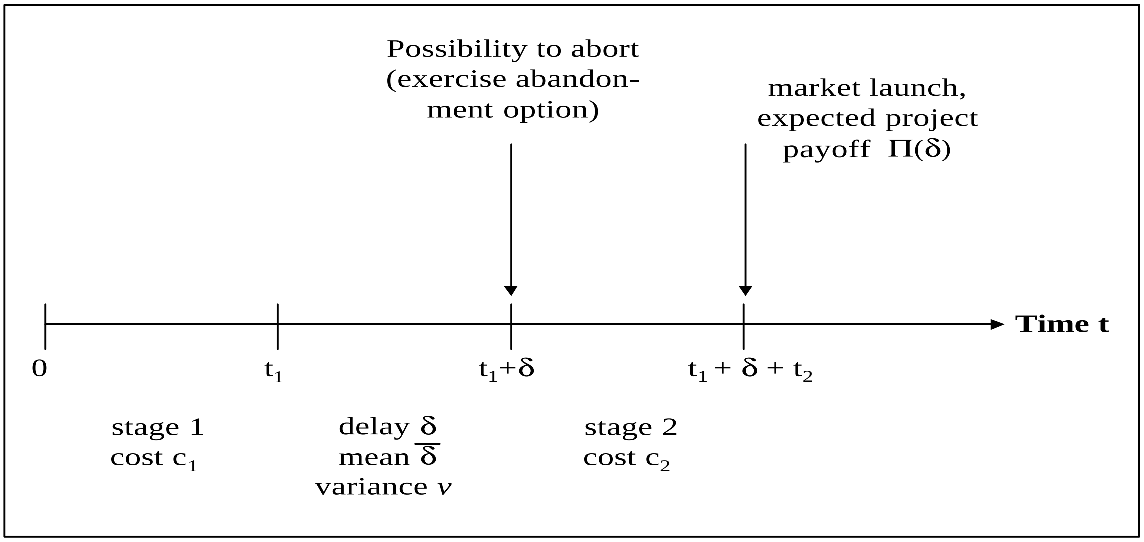

Figure 9: A Development Project With Schedule Variability

The first stage may be interpreted as technical development, with the risk of a delay δ, and the second stage as the marketing and launch campaign. The expected project payoff is a strictly decreasing function of the time-to-market (or, equivalently, of the delay δ). Management may, after the delay has been observed, decide to abort the project before the launch costs are incurred. This possibility to abort represents an abandonment option.

The decision rule at the beginning of stage 2 (at which point δ has been revealed) is clear: continue if $\Pi(\delta) - c_2 > 0$, and abort otherwise. We can invert $\Pi(\delta)$ and thus write the prior probability of continuation as $P\{\delta \leq \Pi^{-1}(c_2)\}$, where Π^{-1} stands for the inverse.

[8]The impact of delays on revenues can be incorporated into the basic model from Section 3 by expanding the state space from (i, t) to (i, t, Δ), where Δ is the accumulated delay up to time t. This would complicate the model without adding clarity to the argument. Similarly, we formally leave discounting out of the model. Discounting alone would correspond to Π decreasing convexely with δ, which is incorporated in our analysis as a special case.

This allows us to write the optimal project value and the NPV of the project (which assumes continuation regardless of the delay). The value of the abandonment option is the difference:

$$V_0(0) = -c_1 + P\{\delta \leq \Pi^{-1}(c_2)\}E[\Pi(\delta) - c_2 \mid \delta \leq \Pi^{-1}(c_2)];$$
$$NPV = -c_1 - c_2 + E[\Pi(\delta)];$$
$$\text{Option value } = P\{\delta > \Pi^{-1}(c_2)\}(c_2 - E[\Pi(\delta) \mid \delta > \Pi^{-1}(c_2)]). \quad (5)$$

As this expression shows, the option value lies in the avoidance of the loss-making case where the reduced payoffs are too small to cover the launch costs. The option value depends on where the critical cutoff delay $\Pi^{-1}(c_2)$ lies with respect to the distribution of δ. This critical delay indicates how“bad” things must become before the project is aborted. If the critical delay is very large, it is very unlikely that revenues are reduced so much as to make the project unprofitable. Thus, the option is unlikely to be exercised and not worth much. If the critical delay is small, the option is likely to be "in the money," and thus worth more.

How does increased schedule uncertainty, represented by the delay variance v^2, influence this option value? The answer depends on two effects: first, the probability of exercising the option is determined by the distribution of the delay δ, and second, the shape of the payoff over time $\Pi(\delta)$ influences the effect of averaging. Proposition 5 summarizes the result.

Proposition 5 *If the critical delay* $\Pi^{-1}(c_2)$ *is small (large) relative to the expected delay* $\overline{\delta}$*, an increasing schedule variability* v^2 *may decrease (increase) the option value of flexibility.*

If the payoff function $\Pi(\delta)$ *is convex (concave), an increasing schedule variability* v^2 *may decrease (increase) the option value of flexibility.*

Two simple examples best illustrate the essence of the argument. First, suppose there is a critical introduction date δ_{crit}, for example, the announced introduction date by a competitor or a regulatory deadline, beyond which revenues suffer discontinuously. $\Pi(\delta)$ is unaffected at $H > c_2$ as long as $\delta < \delta_{crit}$, but it drops to $L < c_2$ if $\delta \geq \delta_{crit}$. Suppose also that the delay is normally distributed with parameters $(\overline{\delta}, v)$. Then the option value (5) becomes $[1 - \Phi((\delta_{crit} - \overline{\delta})/v)](c_2 - L)$. The derivative of this option value with respect to the standard deviation v is $\frac{c_2 - L}{v^2}\phi((\delta_{crit} - \overline{\delta})/v)(\delta_{crit} - \overline{\delta})$. This is positive for $\delta_{crit} > \overline{\delta}$ and negative for $\delta_{crit} < \overline{\delta}$. This means that when the expected delay is large, the project will only be carried through in the left tail of the distribution. This left tail increases with v, so the probability of the option being exercised shrinks with v. As the low payoff L is constant in δ, the probability of exercise determines the value of the option.

The second example shows the effect of averaging over convex functions. A convex $\Pi(\delta)$ corresponds to a situation where a small delay does a lot of damage, but then further delays matter less and less. Suppose the payoff $\Pi(\delta) = 50e^{-0.1\delta}$, and the continuation cost $c_2 = 20$. Then the cutoff delay at which the abandonment is exercised is $\Pi^{-1}(c_2) = 9.16$. Now suppose that the delay is normally distributed with parameters $(30, v)$. We find that with $v = 10$, the option value is 16.1, with $v = 20$ it is 13.9, and with $v = 30$ it shrinks to 12.7. A larger variance of δ spreads the possible delays and thus increases the expected NPV payoff (over the region where the option is exercised), reducing the expected NPV loss. Similarly to technical performance and market requirement variability, schedule uncertainty "smears out" payoff variability against which flexibility has value.

5 Conclusion

In this article, we have developed a simple real option model of an R&D project, where not only the payoff is subject to uncertainty, but also operational variables of budget, technical product performance, required market requirements, and schedule. In each of T stages of the project, management has the flexibility of improving (through a "crash effort") or abandoning the project when additional information becomes available. "Improvement" represents an extra source of option value, in addition to continuation, abandonment, expansion, contraction, or switching. Improvement is the capability of an operational mid-course correction during the execution of the project.

We investigate, in the context of our model, the effect of *operational* variability on the option value of flexibility. We find that operational variability influences the value of flexibility through its influence on the "useable" variability in the project payoff. Budget variability (if subsequent period costs are correlated) increases the useable project payoff variability, and thus the value of flexibility in responding to emerging budget information. However, more variability in product quality may *reduce* the option value when the expected market payoff function is not linear, but convex-concave. Higher quality variability leads to "averaging" over the market payoff function, so flexibility is less useful in avoiding the downside. Similarly, higher market requirement variability results in a part of the payoff range "escaping" beyond the reachable product performance range of the project. Again, the useable payoff variability for the project, and thus the value of managerial flexibility, is diminished. These two results suggest that non-linear payoff functions of the underlying assets as well as the variability ranges of the uncertain variables should be further investigated. Finally, the effect of schedule variability may increase or decrease the option value, depending on the probability of the option being "in the money" and on the convexity of the payoff function.

The results of our model have clear managerial implications, indicating when it is most important to delay commitments. In a mature and predictable project, project management flexibility (the capability of a fast response to emerging events through abandonment and improvement) offer a significant option value. However, in a radically new project with high technical and/or market requirement uncertainty, the option value of flexible project management is reduced. This is consistent with recent findings in the empirical product development literature (e.g., O'Connor 1998, Chandy and Tellis 1998). This may change if the information structure of the project can be altered: if the project manager can gather additional information about product performance (e.g., through testing or simulation), and if the product manager can find out more about customer requirements (e.g., through customer input based on prototypes), it may be valuable to maintain flexibility until the point when this additional information becomes available. Thus, there is an option value of additional information, which can be addressed in the context of the model presented in this article.

If schedule adherence of a project is uncertain and revenues fall off convexely with delay (that is, a large revenue loss is suffered quickly), the value of an abandonment option may diminish if schedule uncertainty grows. That is, the project manager may conclude it is not worthwhile to build in flexibility, but rather abandon the project right away.

The model proposed in this paper is only a qualitative illustration of the fact that different types of variability may have differing effects on real option values. We have not addressed the issues of dynamic R&D investment policies for several R&D projects in parallel, or reduction of market requirement uncertainty over time. Such considerations may lead to additional types of variability with surprising effects. In addition, empirical application of the real options framework in R&D environments is scarce and should be emphasized more. In this article, we emphasize that operational risks influence the real option value of R&D projects. We make a step toward understanding the connection between operational risks and real options. As R&D project costs and risks increase, evaluation of flexibility will become even more important.

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6 Appendix

6.1 Proof of Proposition 1

We proceed by induction. By assumption, Π_i is convex-concave increasing. We show first that the control policy is optimal as claimed. Then we show that the resulting value function $V_i(t)$ is convex-concave increasing as well.

Lemma 1. $V_i(t)$ has the described optimal policy.

Proof. The values corresponding to continuation and improvement in (4) both increase in i because $V_i(t+1)$ does. Thus, if we find an $L_d(t)$ for which abandoning is the best action, abandoning is also best for all $i \leq L_d(t)$. This determines region A in Figure 4. The reader should note that region A is independent of regions C_1, C_2 and I.

Improvement is preferred over continuation in state i iff

$$\alpha(t) < R_i \equiv \frac{\sum_{j=1}^{N}[pV_{i+1+j/2}(t+1)+(1-p)V_{i+1-j/2}(t+1)]}{N(1+r)} - \frac{\sum_{j=1}^{N}[pV_{i+j/2}(t+1)+(1-p)V_{i-j/2}(t+1)]}{N(1+r)}. \tag{6}$$

By convex-concavity of $V_i(t+1)$, the right-hand side (rhs) of (6) first increases, then decreases in i. Thus, if there is an $L_m(t)$ with $R_{L_m(t)} < R_{L_m(t)+1}$ such that $\alpha(t) \geq R_{L_m(t)}$ but $\alpha(t) \leq R_{L_m(t)+1}$, continuation is preferred for all $i \leq L_m(t)$. For state $L_m(t)+1$, improvement is preferred. This describes region C_1 in Figure 4.

If there is an $L_u(t)$ with $R_{L_u(t)} < R_{L_u(t)-1}$ (i.e., $V_i(t)$ is locally concave), such that $\alpha(t) \geq R_{L_u(t)}$ but $\alpha(t) \leq R_{L_u(t)-1}$, then continuation is preferred, for all $i \geq L_u(t)$. For state $L_u(t)-1$, improvement is preferred. This determines region C_2 in Figure 4.

Finally, by convex-concavity of $V_i(t+1)$, there can be no additional switch of condition (6) in between, which settles region I in Figure 4. □

Lemma 2. $V_i(t)$ is convex-concave increasing in i.

Proof. Within the regions of Figure 4, $V_i(t)$ is convex-concave increasing since it is a linear combination of summands from $V_i(t+1)$. It remains to check the borders between the regions.

At $i = L_u(t)+1$, it is optimal to choose continuation, and at i it is optimal to choose improvement.

$$\begin{aligned}
V_{i+1}(t) - V_i(t) &= \alpha(t); \\
V_i(t) - V_{i-1}(t) &= \frac{\sum_{j=1}^{N}[pV_{i+1+j/2}(t+1)+(1-p)V_{i+1-j/2}(t+1)]}{N(1+r)} \\
&\quad - \frac{\sum_{j=1}^{N}[pV_{i+j/2}(t+1)+(1-p)V_{i-j/2}(t+1)]}{N(1+r)} \\
&\geq \alpha(t) \text{ by Equation (6).}
\end{aligned}$$

Thus, $V_i(t)$ is concave at $L_u(t)$. A symmetric argument at $i = L_m(t)$ implies that $V_i(t)$ is convex at $L_m(t)$.

Finally, for $L_d(t)$ we must consider two cases. First, if $L_d(t)$ is in the convex region of $V_i(t)$, we can write the increments of $V_i(t)$ as follows:

$$\begin{aligned}
V_{L_d(t)-1}(t) = V_{L_d(t)}(t) = &\ 0 \text{ by definition of } L_d(t); \text{ thus:} \\
V_{L_d(t)+1}(t) - V_{L_d(t)}(t) = &\ \frac{p\sum_{L_d(t)+3/2}^{L_d(t)+(N+2)/2} V_j(t+1) + (1-p)\sum_{L_d(t)+1/2}^{L_d(t)-(N-2)/2} V_j(t+1)}{N(1+r)} \\
&- \frac{p\sum_{i+1/2}^{i+N/2} V_j(t+1) + (1-p)\sum_{i-1/2}^{i-N/2} V_j(t+1)}{N(1+r)} - 0 \\
\leq &\ V_{L_d(t)+2}(t) - V_{L_d(t)+1}(t) \text{ because } V_{L_d(t)}(t) \text{ is bounded below at zero} \\
&\text{and by local convexity of } V_i(t+1).
\end{aligned}$$

Thus, $V_i(t)$ is convex at $L_d(t)$. Second, if $L_d(t)$ is in the concave region of $V_i(t)$, the symmetric argument can be used to establish that $V_i(t)$ is concave at $L_d(t)$. □

6.2 Proof of Proposition 2

Consider two payment distributions (m, M) and $(\overline{m}, \overline{M})$ such that $(\overline{M} - \overline{m}) > (M - m)$, but the averages are equal. Denote with the upper bar all policies and results corresponding to $(\overline{m}, \overline{M})$. Then $\overline{\Pi}_i > \Pi_i$ for all $i > ED = 0$ and *vice versa*. Moreover, $\overline{\Pi}_i - \overline{\Pi}_{i-1} \geq \Pi_i - \Pi_{i-1}$ for all i, by the definition of the payoff function. Thus, the two payoff function fulfill the conditions for the proposition.

Lemma 3. Suppose there are two value functions $V_i(t+1)$ and $\overline{V}_i(t+1)$, both convex-concave increasing in i, with the following characteristics:

$$\text{larger increments: } \overline{V}_i(t+1) - \overline{V}_{i-1}(t+1) \geq V_i(t+1) - V_{i-1}(t+1)\ \forall i; \tag{7}$$

$$\text{equal value: } \sum_{i_{min}(t+1)}^{i_{max}(t+1)} \overline{V}_i(t+1) \geq \sum_{i_{min}(t+1)}^{i_{max}(t+1)} V_i(t+1), \tag{8}$$

where i_{min} is the lowest performance possible at launch if continuation is chosen in all project states, and i_{max} is the highest. Then $V_i(t)$ and $\overline{V}_i(t)$ are convex-concave increasing and fulfill conditions (7) and (8).

Proof. Convex-concavity of both value functions follows directly from Proposition 1. Each value function is, by its definition (4), a linear combination within the regions of improvement (I) and continuation (C) separately. This implies condition (7) for the regions I and C separately. In addition, by the definition (6) of L_m and L_u, $\overline{L}_m \leq L_m$, and $\overline{L}_u \geq L_u$, that is, the range of action I is larger for $\overline{V}_i(t)$ because $\overline{V}_i(t+1)$ is steeper by condition (7).

At the transition $\overline{L}_m$, where $\overline{V}_i(t)$ switches to I while $V_i(t)$ still stays with C, condition (7) also holds because the expected improvement payoff more than makes up for additional improvement costs α_t. Similarly, at the upper transition L_u, $V_i(t)$ switches back to C while $\overline{V}_i(t)$ still stays with I because $\overline{V}_i(t+1)$ is still steep enough to justify the improvement cost, while $V_i(t+1)$ is not. Thus condition (7) holds here as well. Finally, at the abandonment control L_d, we can argue that $\overline{V}_i(t)$ is prevented from "dipping below zero," which limits its disadvantage where it is below $V_i(t)$. (The algebraic details of these comparisons are left to the reader). This establishes condition (7) for $\overline{V}_i(t)$ and $V_i(t)$.

To see that condition (8) holds, recall that if continuation was chosen everywhere, (8) would hold for time t because both period t value functions are symmetric linear combinations of the period $t+1$ value functions. But as the improvement range of $\overline{V}_i(t)$ is enlarged, condition (6) ensures that the improvement enhances $\overline{V}_i(t)$ at least for some states, and similarly, abandonment limits $\overline{V}_i(t)$ from below. Therefore, (8) holds for time t. □

The proposition can now be proved by induction: $\overline{\Pi}_i$ and Π_i are convex-concave increasing and fulfill conditions (7) and (8) by assumption, crossing over at $i = 0$. Then Lemma 3 establishes an induction backwards from T to time 0, and since at time $t = 0$ there is only one state, $\overline{V}_0(0) \geq V_0(0)$.

At the same time, both payoff functions $\overline{\Pi}_i$ and Π_i have the same project NPV (associated with continuation chosen at every state). The reason is that the compounded probability distribution of the payoffs (because $p = 0.5$), as well as the differences $\overline{\Pi}_i - \Pi_i$ (by assumption), are symmetric around zero. Therefore, the option value of flexibility from improvement and abandonment, which corresponds to the difference $V_0(0) - NPV$, is larger for $\overline{\Pi}_i$. □

6.3 Proof of Proposition 3

We include N as an explicit parameter in the value function $V_{T,N}(i)$. We prove that for every N, there exists an i^*_N such that $V_{T,N+1}(i) \geq V_{T,N}(i)$ for all $i < i^*_N$ and $V_{T,N+1}(i) \leq V_{T,N}(i)$ for all $i \geq i^*_N$. That is, the value function increases with the technical uncertainty N below an inflection point and decreases with N above the inflection point. As a result, the value function $V_{T,N}(i)$ is "squeezed" more closely and has thus smaller increments. Therefore, by Proposition 2, the option value $V_0(0)$ decreases in N, reflecting the reduced potential for risk hedging. Figure 5 in the body of the text summarizes the "intuition" of this argument.

First, consider the expected payoff in period T from continuation. From Equation (3) and $p = 0.5$,

$$V_{T,N}(i)(\text{cont.}) = -c(T) + \frac{\sum_{j=1}^{N} \Pi_{i+j/2} + \Pi_{i-j/2}}{N(1+r)}.$$

Convex-concavity of Π_i implies that at $i = 0$, the first summand in the numerator decreases with N, and the second summand in the numerator increases with N. As $i > 0$, the convex combination in the numerator shifts more toward the concave part of Π_i and thus toward decreasing in N, and *vice versa*. Therefore, we can define $i_{cont.}(T)$ analogously to Proposition 2 such that $V_{T,N}(i)(\text{cont.})$ increases in N for all $i \leq i_{cont.}(T)$ and $V_{T,N}(i)(\text{cont.})$ decreases in N for all $i > i_{cont.}(T)$. Moreover, by symmetry of Π_i, $i_{cont.}(T) = 0$.

With this argument, we can show that there exists an $i_{impr.}(N)$ such that $V_{T,N}(i)(\text{impr.})$ (defined in the same way as $V_{T,N}(i)(\text{cont.})$ above) increases in N for all $i \leq i_{impr.}(N)$ and $V_{T,N}(i)(\text{impr.})$ decreases in N for all $i > i_{impr.}(N)$. Moreover, $i_{impr.}(N) = i_{cont.}(N) - 1 = -1$, which can easily be seen from the fact that the two expected payoffs are only shifted by one performance level.

By convex-concavity of Π_i and Equation (6), $L_m(T, N)$ must increase in N, and $L_u(T, N)$ must decrease in N. Therefore, when considering two technical variability levels $N_1 < N_2$, we find that the two corresponding value functions fit the structure in Figure 7 with $V_{T,N_1}(i)$ corresponding to the higher variability value function $\overline{V}_T(i)$ in Lemma 3. Proposition 2, therefore, implies that the option value $V_0(0)$ decreases in N. □

6.4 Proof of Proposition 4

Consider two performance requirement distributions with equal mean ED but $\overline{\sigma} > \sigma$. Denote with the upper bar all policies and results corresponding to the distribution with the larger standard deviation $\overline{\sigma}$. This implies that the payoff function $\overline{\Pi}_i$ has the same mean but *lower* variability: $(\overline{\Pi}_i - \overline{\Pi}_{i-1}) < (\Pi_i - \Pi_{i-1})$ such that $(\overline{\Pi}_i - \Pi_i) < 0$ for $i > ED$ and vice versa for $i < ED$. Therefore, Proposition 2 applies with Π_i and $\overline{\Pi}_i$ exchanged. This proves statements 1 and 2 of the proposition.

Finally, suppose that $\overline{\Pi}_{ED}/(1+r)^T > \sum_{t=1}^{T-1} c(t)$, that is, the project exceeds its variable costs in expectation. There exists a σ^* such that $(\overline{\Pi}_i - \overline{\Pi}_{i-1})/(1+r) < \alpha(t)$ for all $\overline{\sigma} > \sigma^*$ and all (i,t) and $\overline{\Pi}_{-NT/2}/(1+r)^T > \sum_{t=0}^{T-1} c(t)/(1+r)^t$. Then continuation will be chosen in all states (i,t) since the payoff increments are too small to make improvement worthwhile and the payoff is still high enough even in the worst reachable state to permit continuation.

□